Sufficiency of the solution for multi-objective simi-infinite programming with K – (F_b, ρ) – convexity

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Abstract

In this paper, some nonsmooth generalized convex functions called uniform K – (F_b, ρ) – convex function, uniform K – (F_b, ρ) – pseudoconvex function, uniform K – (F_b, ρ) – quasiconvex function are defined using K – directional derivative and K – subdifferential. Nonsmooth multi-objective semi-infinite programming involving these generalized convex functions is researched, some sufficient optimality conditions are obtained.

Keywords: nonsmooth, multi-objective semi-infinite programming, sufficient optimality conditions, uniform K – (F_b, ρ) – convex function

1 Introduction

The convexity theory plays an important role in many aspects in mathematical programming. In recent years, to relax convexity assumption involved in sufficient conditions for optimality or duality theorems, various generalizations of convex functions have appeared in the literature. Hanson and Mond introduced type I and type II function[1]. Reuda and Hanson extended type I function and obtained pseudo type I and quasi type I function [2]. Bector and Singh introduced b – convex function [3]. Bector, Suneja and Cupta extended b – convex function and defined univex function [4]. Mishra discussed the optimality and duality for multi-objective programming with generalized univexity [5]. Preda introduced (F_b, ρ) – convex function as extension of F – convex function and ρ – convex function [6-8]. Aghezaf and Hachimi discussed the sufficiency and duality for multi-objective programming involving generalized (F_b, ρ) – convexity [9].

In this paper, we introduce a new classes of generalized convex functions, that is, uniform K – (F_b, ρ) – convex function, uniform K – (F_b, ρ) – pseudoconvex function, uniform K – (F_b, ρ) – quasiconvex function, etc. Then we consider nonsmooth multi-objective semi-infinite programming involving these generalized convex functions and obtain some sufficient optimality conditions.

2 Definitions

Throughout this paper, let R^n be the $n-$dimensional Euclidean space and R^n be its non-negative orthat. Now we consider the following multi-objective semi-infinite programming problem:

$$(VP)\begin{cases}\min f(x)=(f_1(x), f_2(x),..., f_p(x))\\ s.t. g(x,u)\le 0, x\in X, u\in U\end{cases},$$

where X is an open subset of R^n, $f: X\to R^p$, $g: X\times U\to R^n$, $U\subset R$ is an infinite parameter set. Let $X^0=\{x\,|\,g(x,u)\le 0, x\in X, u\in U\}, \Delta=\{i\,|\,g(x,u^i)\le 0, x\in X, u^i\in U\}$, $I(x^*)=\{i\,|\,g(x^*,u^i)\le 0, x^*\in X, u^i\in U\}$, $U^*=\{u^i\in U\,|\,g(x,u^i)\le 0, x\in X, i\in\Delta\}$ is any countable subset of U, $\Lambda=\{\mu_j\,|\,\mu_j\ge 0|, j\in\Delta,$ there is only finite μ_j such that $\mu_j\ne 0\}$.

Notations. If $x,y\in R^n$, then $x\le y\Leftrightarrow x_i\le y_i$, $i=1,2,\cdots,$ n and there exists at least one $i_0\in\{1,2,\cdots,n\}$ such that $x_{i_0}<y_{i_0}$; $x<y\Leftrightarrow x_i<y_i, i=1,2,\cdots,n.$

Definition 1 [10]: Let $K(\cdot,\cdot)$ is a local cone approximation, the function $f^K(x,\cdot): X\to R$ with $f^K(x;y):=\inf\{\xi\in R\,|\,(y,\xi)\in K(epif,(x,f(x)), y\in R^n)\}$ is called $K-$directional derivative of f at x.

Definition 2 [10]: A function $f: X\to R$ is called $K-$subdifferentiable at x if there exists a convex compact set $\partial^K f(x)$ such that $f^K(x,y)=\max_{\xi\in\partial^K f(x)}<\xi,y>,\forall y\in R^n$, where $\partial^K f(x):=\{x^*\in X^*\,|<y,x^*>\le f^K(x;y),\forall y\in R^n\}$ is called $K-$subdifferential of f at x.

Definition 3: A functional $F: X\times X\times R^n\to R$ ($X\subset R^n$) is called sublinear with respect to the third variable, if for any $x_1, x_2\in X$:

(i) $F(x_1,x_2;a_1+a_2)\le F(x_1,x_2;a_1)+F(x_1,x_2;a_2),\forall a_1,a_2\in R^n;$

(ii) $F(x_1,x_2;ra)=rF(x_1,x_2;a),\forall r\in R, r\ge 0, a\in R^n.$

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***Definition 4*: ** $x^* \subset X^0$ is called an efficient solution for (VP) if and only if there exists no $x \subset X^0$ such that $f(x^*) \le f(x)$.

***Definition 5*: ** $x^* \subset X^0$ is called a weak efficient solution for (VP) if and only if there exists no $x \subset X^0$ such that $f(x^*) < f(x)$.

In the following definitions, we suppose $C \subset R^n$ is a nonempty set, $x_0 \in C$, $f: C \to R$ is a local Lipschitz function at x_0, $F: C \times C \times R^n \to R$ is sublinear with respect to the third variable, $\phi: R \to R$, $b: C \times C \times [0,1] \to R_+$, $\lim\limits_{\lambda \to 0^+} b(x, x_0; \lambda) = b(x, x_0)$, $d(\cdot,\cdot)$ is a pseudo-metric in R^n. In [9], Elster and Thierfelder defined $K-$directional derivative and $K-$subdifferential and pointed out that $K-$subdifferential is most generalized. Now we will define some new generalized convex functions using $K-$directional derivative and $K-$subdifferential.

***Definition 6*: ** A function $f: C \to R$ is said to be uniform $K-(F_b, \rho)-$convex at x_0 with respect to F, ϕ, b, d, if for all $x \in C$, there exists $\rho \in R$ such that

$$b(x,x_0)\phi[f(x) - f(x_0)] \ge F(x, x_0; \xi) + \rho d^2(x,y), \forall \xi \in \partial^K f(x_0).$$

***Definition 7*: ** A function $f: C \to R$ is said to be strictly uniform $K-(F_b, \rho)-$ convex at x_0 with respect to F, ϕ, b, d, if for all $x \in C, x \ne x_0$, there exists $\rho \in R$ such that

$$b(x,x_0)\phi[f(x) - f(x_0)] > F(x, x_0; \xi) + \rho d^2(x,x_0), \forall \xi \in \partial^K f(x_0).$$

***Definition 8*: ** A function $f: C \to R$ is said to be uniform $K-(F_b, \rho)-$ pseudoconvex at x_0 with respect to F, ϕ, b, d, if for all $x \in C$, there exists $\rho \in R$ such that

$$b(x,x_0)\phi[f(x) - f(x_0)] < 0 \Rightarrow,$$

$$F(x, x_0; \xi) + \rho d^2(x,y) < 0, \forall \xi \in \partial^K f(x_0).$$

***Definition 9*: ** A function $f: C \to R$ is said to be strictly uniform $K-(F_b, \rho)-$pseudoconvex at x_0 with respect to F, ϕ, b, d, if for all $x \in C, x \ne x_0$, there exists $\rho \in R$ such that:

$$b(x,x_0)\phi[f(x) - f(x_0)] \le 0 \Rightarrow,$$

$$F(x, x_0; \xi) + \rho d^2(x,y) < 0, \forall \xi \in \partial^K f(x_0).$$

***Definition 10*: ** A function $f: C \to R$ is said to be uniform $K-(F_b, \rho)-$quasiconvex at x_0 with respect to F, ϕ, b, d, if for all $x \in C$, there exists $\rho \in R$ such that:

$$b(x,x_0)\phi[f(x) - f(x_0)] \le 0 \Rightarrow,$$

$$F(x, x_0; \xi) + \rho d^2(x,y) \le 0, \forall \xi \in \partial^K f(x_0).$$

***Definition 11*: ** A function $f: C \to R$ is said to be weak uniform $K-(F_b, \rho)-$quasiconvex at x_0 with respect to F, ϕ, b, d, if for all $x \in C$, there exists $\rho \in R$ such that:

$$b(x,x_0)\phi[f(x) - f(x_0)] < 0 \Rightarrow,$$

$$F(x, x_0; \xi) + \rho d^2(x,y) \le 0, \forall \xi \in \partial^K f(x_0).$$

3 Sufficient optimality conditions

In this section, we obtain some sufficient conditions for a feasible x^* to be efficient or weak efficient for (VP) in the form of the following theorems.

***Theorem 1*: ** Assume that $x^* \in X^0$, if for any $x \in X^0$, there exist $F, \phi_1, \phi_2, b_1, b_2, \rho_1 \in R^p$, $\rho_2 \in R^{|I|}$, $\lambda^* > 0$, $\sum_{i=1}^{p} \lambda_i^* = 1$, $\mu_j^* \in \Lambda$, $j \in I(x^*)$, such that:

(i) $f_i(x) (i = 1, 2, \cdots, p)$ is uniform $K-(F_{b_1}, \rho_1^i)-$convex at x^*;

(ii) $g(x, u^j) (j \in I(x^*))$ is uniform $K-(F_{b_2}, \rho_2^j)-$convex at x^*;

(iii) $0 \in \sum_{i=1}^{p} \lambda_i^* \partial^K f_i(x^*) + \sum_{j \in I(x^*)} \mu_j^* \partial^K g(x^*, u^j), \ \forall u^j \in U^*$;

(iv) $\alpha < 0 \Rightarrow \phi_1(\alpha) < 0, \phi_1(0) = 0, \alpha \le 0 \Rightarrow \phi_2(\alpha) \le 0$, $b_1(x, x^*) > 0, b_2(x, x^*) \ge 0$;

(v) $\sum_{i=1}^{p} \lambda_i^* \rho_1^i + \sum_{j \in I(x^*)} \mu_j^* \rho_2^j \ge 0$.

Then x^* is an efficient solution for (VP).

Proof: Suppose that x^* is not an efficient solution for (VP), then there exists $\bar{x} \in X$ and at least one $i_0 \in \{1, 2, \cdots, p\}$ such that:

$$f_{i_0}(\bar{x}) - f_{i_0}(x^*) < 0,$$

$$f_i(\bar{x}) - f_i(x^*) \le 0, i \ne i_0.$$

By hypothesis (iv), we have:

$$b_1(\bar{x}, x^*) \phi_1[f_i(\bar{x}) - f_i(x^*)] \le 0, i = 1, 2, \cdots, p.$$

and there exists at least one inequality which is a strict inequality.

Since $\lambda_i^* > 0, i = 1, 2, \cdots, p,$ we have

$$\sum_{i=1}^{p} \lambda_i^* F(\bar{x}, x^*; \xi_i) + \sum_{i=1}^{p} \lambda_i^* \rho_1^i d^2(\bar{x}, x^*) < 0. \quad (1)$$

By hypothesis (iii), there exists $\xi_i^* \in \partial^K f_i(x^*), i = 1, 2, \cdots, p$ and $\eta_j^* \in \partial^K g(x^*, u^j), j \in I(x^*)$ such that:

$$\sum_{i=1}^{p} \lambda_i^* \xi_i^* + \sum_{j \in I(x^*)} \mu_j^* \eta_j^* = 0.$$

so:

$$F\left(\bar{x}, x^*; \sum_{i=1}^{p} \lambda_i^* \xi_i^* + \sum_{j \in I(x^*)} \mu_j^* \eta_j^*\right) = F(\bar{x}, x^*; 0) = 0. \qquad (2)$$

Observe that $g(\bar{x}, u^j) \le 0 = g(x^*, u^j), j \in I(x^*),$ we have:

$g(\bar{x}, u^j) - g(x^*, u^j) \le 0, j \in I(x^*).$

By hypothesis (iv), we have:

$b_2(\bar{x}, x^*)\phi_2[g(\bar{x}, u^j) - g(x^*, u^j)] \le 0, j \in I(x^*).$

By hypothesis (ii), we get

$F(\bar{x}, x^*; \eta_j) + \rho_2^j d^2(\bar{x}, x^*) \le 0, \forall \eta_j \in \partial^K g(x^*, u^j).$

since $\mu_j^* \ge 0$, we have:

$$\sum_{j \in I(x^*)} \mu_j^* F(\bar{x}, x^*; \eta_j) + \sum_{j \in I(x^*)} \mu_j^* \rho_2^j d^2(\bar{x}, x^*) \le 0. \qquad (3)$$

Adding Equations (1) and (3), using the sublinearity of F, we can obtain:

$$F\left(\bar{x}, x^*; \sum_{i=1}^{p} \lambda_i^* \xi_i^* + \sum_{j \in I(x^*)} \mu_j^* \eta_j^*\right) + \left(\sum_{i=1}^{p} \lambda_i^* \rho_1^i + \sum_{j \in I(x^*)} \mu_j^* \rho_2^j\right) d^2(\bar{x}, x^*) < 0,$$

by hypothesis (v), we have

$$\sum_{i=1}^{p} \lambda_i^* \rho_1^i + \sum_{j \in I(x^*)} \mu_j^* \rho_2^j \ge 0.$$

so

$$F\left(\bar{x}, x^*; \sum_{i=1}^{p} \lambda_i^* \xi_i^* + \sum_{j \in I(x^*)} \mu_j^* \eta_j^*\right) < 0,$$

which contradicts Equation (2). Therefore, x^* is an efficient solution for (VP).

***Theorem 2*:** Assume that $x^* \in X^0$, if for any $x \in X^0$, there exist $F, \phi_1, \phi_2, b_1, b_2, \rho_1 \in R^p$, $\rho_2 \in R^{|I|}$, $\lambda^* \ge 0$, $\sum_{i=1}^{p} \lambda_i^* = 1$, $\mu_j^* \in \Lambda$, $j \in I(x^*)$, such that:

(i) $f_i(x)(i = 1, 2, \cdots, p)$ is uniform $K - (F_{b_1}, \rho_1^i) -$ convex at x^*;

(ii) $g(x, u^j)(j \in I(x^*))$ is uniform $K - (F_{b_2}, \rho_2^i) -$ convex at x^*;

(iii) $0 \in \sum_{i=1}^{p} \lambda_i^* \partial^K f_i(x^*) + \sum_{j \in I(x^*)} \mu_j^* \partial^K g(x^*, u^j),\ \forall u^j \in U^*$;

(iv) $\alpha < 0 \Rightarrow \phi_1(\alpha) < 0, \phi_1(0) = 0, \alpha \le 0 \Rightarrow \phi_2(\alpha) \le 0,$ $b_1(x, x^*) > 0, b_2(x, x^*) \ge 0$;

(v) $\sum_{i=1}^{p} \lambda_i^* \rho_1^i + \sum_{j \in I(x^*)} \mu_j^* \rho_2^j \ge 0.$

Then x^* is a weak efficient solution for (VP).

***Theorem 3*:** Assume that $x^* \in X^0$, if for any $x \in X^0$, there exist $F, \phi_1, \phi_2, b_1, b_2, \rho_1 \in R^p$, $\rho_2 \in R^{|I|}$, $\lambda^* \ge 0$, $\sum_{i=1}^{p} \lambda_i^* = 1$, $\mu_j^* \in \Lambda$, $j \in I(x^*)$, such that:

(i) $f_i(x)(i = 1, 2, \cdots, p)$ is strictly uniform $K - (F_{b_1}, \rho_1^i) -$ pseudoconvex at x^*;

(ii) $g(x, u^j)(j \in I(x^*))$ is uniform $K - (F_{b_2}, \rho_2^j) -$ quasi convex at x^*;

(iii) $0 \in \sum_{i=1}^{p} \lambda_i^* \partial^K f_i(x^*) + \sum_{j \in I(x^*)} \mu_j^* \partial^K g(x^*, u^j),\ \forall u^j \in U^*$;

(iv) $\alpha < 0 \Rightarrow \phi_1(\alpha) < 0, \phi_1(0) = 0, \alpha \le 0 \Rightarrow \phi_2(\alpha) \le 0,$ $b_1(x, x^*) > 0, b_2(x, x^*) \ge 0$;

(v) $\sum_{i=1}^{p} \lambda_i^* \rho_1^i + \sum_{j \in I(x^*)} \mu_j^* \rho_2^j \ge 0.$

Then x^* is an efficient solution for (VP).

***Theorem 4*:** Assume that $x^* \in X^0$, if for any $x \in X^0$, there exist $F, \phi_1, \phi_2, b_1, b_2, \rho_1 \in R^p$, $\rho_2 \in R^{|I|}$, $\lambda^* \ge 0$, $\sum_{i=1}^{p} \lambda_i^* = 1$, $\mu_j^* \in \Lambda$, $j \in I(x^*)$, such that:

(i) $f_i(x)(i = 1, 2, \cdots, p)$ is uniform $K - (F_{b_1}, \rho_1^i) -$ pseudoconvex at x^*;

(ii) $g(x, u^j)(j \in I(x^*))$ is uniform $K - (F_{b_2}, \rho_2^i) -$ quasiconvex at x^*;

(iii) $0 \in \sum_{i=1}^{p} \lambda_i^* \partial^K f_i(x^*) + \sum_{j \in I(x^*)} \mu_j^* \partial^K g(x^*, u^j),\ \forall u^j \in U^*$;

(iv) $\alpha < 0 \Rightarrow \phi_1(\alpha) < 0, \phi_1(0) = 0, \alpha \le 0 \Rightarrow \phi_2(\alpha) \le 0,$ $b_1(x, x^*) > 0, b_2(x, x^*) \ge 0$;

(v) $\sum_{i=1}^{p} \lambda_i^* \rho_1^i + \sum_{j \in I(x^*)} \mu_j^* \rho_2^j \ge 0.$

Then x^* is a weak efficient solution for (VP).

***Theorem 5*:** Assume that $x^* \in X^0$, if for any $x \in X^0$, there exist $F, \phi_1, \phi_2, b_1, b_2, \rho_1 \in R^p$, $\rho_2 \in R^{|I|}$, $\lambda^* \ge 0$,

$\sum_{i=1}^{p}\lambda_i^*=1,\ \mu_j^*\in\Lambda,\ j\in I(x^*)$ and μ_j^* not all is zero such that:

(i) $f_i(x)(i=1,2,\cdots,p)$ is uniform $K-(F_{b_1},\rho_1^i)-$ quasiconvex at x^*;

(ii) $g(x,u^j)(j\in I(x^*))$ is strictly uniform $K-(F_{b_2},\rho_2^i)-$ pseudoconvex at x^*;

(iii) $0\in\sum_{i=1}^{p}\lambda_i^*\partial^K f_i(x^*)+\sum_{j\in I(x^*)}\mu_j^*\partial^K g(x^*,u^j),\ \forall u^j\in U^*$;

(iv) $\alpha<0\Rightarrow\phi_1(\alpha)<0,\phi_1(0)=0,\alpha\le 0\Rightarrow\phi_2(\alpha)\le 0,$ $b_1(x,x^*)>0,b_2(x,x^*)\ge 0$;

(v) $\sum_{i=1}^{p}\lambda_i^*\rho_1^i+\sum_{j\in I(x^*)}\mu_j^*\rho_2^j\ge 0.$

Then x^* is an efficient solution for (VP).

Theorem 6: Assume that $x^*\in X^0$, if for any $x\in X^0$, there exist $F,\phi_1,\phi_2,b_1,b_2,\rho_1\in R^p,\ \rho_2\in R^{|I|},\ \lambda^*\ge 0,$ $\sum_{i=1}^{p}\lambda_i^*=1,\ \mu_j^*\in\Lambda,\ j\in I(x^*)$ and μ_j^* not all is zero such that:

(i) $f_i(x)(i=1,2,\cdots,p)$ is weak uniform $K-(F_{b_1},\rho_1^i)-$ quasiconvex at x^*;

(ii) $g(x,u^j)(j\in I(x^*))$ is strictly uniform $K-(F_{b_2},\rho_2^i)-$ pseudoconvex at x^*;

(iii) $0\in\sum_{i=1}^{p}\lambda_i^*\partial^K f_i(x^*)+\sum_{j\in I(x^*)}\mu_j^*\partial^K g(x^*,u^j),\ \forall u^j\in U^*$;

(iv) $\alpha<0\Rightarrow\phi_1(\alpha)<0,\phi_1(0)=0,\alpha\le 0\Rightarrow\phi_2(\alpha)\le 0,$ $b_1(x,x^*)>0,b_2(x,x^*)\ge 0$;

(v) $\sum_{i=1}^{p}\lambda_i^*\rho_1^i+\sum_{j\in I(x^*)}\mu_j^*\rho_2^j\ge 0.$

Then x^* is a weak efficient solution for (VP).

Theorem 7: Assume that $x^*\in X^0$, if for any $x\in X^0$, there exist $F,\phi_1,\phi_2,b_1,b_2,\rho_1\in R^1,\ \rho_2\in R^{|I|},\ \lambda^*>0,$ $\sum_{i=1}^{p}\lambda_i^*=1,\ \mu_j^*\in\Lambda,\ j\in I(x^*)$ such that

(i) $\left(\sum_{i=1}^{n}\lambda_i^* f_i\right)(x)$ is uniform $K-(F_{b_1},\rho_1)-$pseudoconvex at x^*;

(ii) $g(x,u^j)(j\in I(x^*))$ is uniform $K-(F_{b_2},\rho_2^i)-$ quasiconvex at x^*;

(iii) $0\in\partial^K\left(\sum_{i=1}^{p}\lambda_i^* f_i)(x^*\right)+\sum_{j\in I(x^*)}\mu_j^*\partial^K g(x^*,u^j),\ \forall u^j\in U^*$;

(iv) $\alpha<0\Rightarrow\phi_1(\alpha)<0,\phi_1(0)=0,\alpha\le 0\Rightarrow\phi_2(\alpha)\le 0,$ $b_1(x,x^*)>0,b_2(x,x^*)\ge 0$;

(v) $\rho_1+\sum_{j\in I(x^*)}\mu_j^*\rho_2^j\ge 0.$

Then x^* is an efficient solution for (VP).

Theorem 8: Assume that $x^*\in X^0$, if for any $x\in X^0$, there exist $F,\phi_1,\phi_2,b_1,b_2,\rho_1\in R^1,\ \rho_2\in R^{|I|},\ \lambda^*>0,$ $\sum_{i=1}^{p}\lambda_i^*=1,\ \mu_j^*\in\Lambda,\ j\in I(x^*)$ and μ_j^* not all is zero such that

(i) $(\sum_{i=1}^{n}\lambda_i^* f_i)(x)$ is weak uniform $K-(F_{b_1},\rho_1^i)-$ quasi convex at x^*;

(ii) $g(x,u^j)(j\in I(x^*))$ is strictly uniform $K-(F_{b_2},\rho_2^i)-$ pseudoconvex at x^*;

(iii) $0\in\partial^K(\sum_{i=1}^{p}\lambda_i^* f_i)(x^*)+\sum_{j\in I(x^*)}\mu_j^*\partial^K g(x^*,u^j),\ \forall u^j\in U^*$;

(iv) $\alpha<0\Rightarrow\phi_1(\alpha)<0,\phi_1(0)=0,\alpha\le 0\Rightarrow\phi_2(\alpha)\le 0,$ $b_1(x,x^*)>0,b_2(x,x^*)\ge 0$;

(v) $\rho_1+\sum_{j\in I(x^*)}\mu_j^*\rho_2^j\ge 0.$

Then x^* is an efficient solution for (VP).

The proofs of Theorems 2-8 are similar to that of Theorem 1.

4 Conclusions

In this paper, we introduce a new classes of generalized convex functions, that is, uniform K – (F_b, ρ) – convex function, uniform K – (F_b, ρ) – pseudoconvex function, uniform K – (F_b, ρ) – quasiconvex function, etc. Then we consider nonsmooth multi-objective semi-infinite programming involving these generalized convex functions and obtain some sufficient optimality conditions.

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